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“How to Talk About Ideas You Don’t Understand”

Thank you Dean Conrad, and to the class of 2014: Welcome to Pomona College. We are excited to have you here. We will also be excited when you leave in four years, but that is mainly because one month after your departure, the next soccer world cup will begin. Let me start by saying that we are confronted by ideas we don't understand all the time. Whether it is the culture of the societies that we invade, the latest study in the health hazards of certain foods, the reasons for the latest TV fads, possible solutions to our environmental woes, or the arguments about social security or health care policy, we are surrounded by ideas that we can certainly discuss, but we don't really understand.

As a result, we end up depending on experts and, often, we are in no position to even question the experts intelligently. And we don't even get the experts' opinions directly. They are often mediated by big corporations, the media, the government, Facebook, Google and other entities with their own agendas. We are constantly bombarded by images, advertisements, games, commentaries, reality shows, blogs, top ten lists and expert opinion of all sorts that not only want to shape our political views, but want to help us understand what kind of a vacation we want, what kind of medicine we should ask our doctors for, what books we should read, how we should look, what GPS features we really need, and even how to be unique.

We are then asked--in an interesting example of a feedback loop--to review and rank our experiences. As a result, we play a role in further homogenizing our collective existence, and further justifying the rationale behind the ideas that we were fed. Oscar Wilde famously said "Be yourself, everyone else is already taken." That is actually very difficult to do. We are all becoming everyone else.

OK. Now that I got an Oscar Wilde quote in, I can relax. An Oscar Wilde quote is a requirement for these talks. Another requirement is to wear socks. I figured that one out of two will keep me out of trouble.

I certainly exaggerated, over-simplified, wasn't too original, and jumped from one thing to another. These are all symptoms of talking about ideas that you don't fully grasp. Last year's Green movement in Iran probably serves as a good counter-example to much of what I said.

We can certainly agree that there are lots of complicated ideas that we don't understand. On a more somber note, I want to point out that even some very basic concepts that we think we understand and we use all the time, are more complicated than first meets the eye.

I will start with something that we all do understand.

What is the area of a rectangle? If I have a 3 by 4 rectangle, then the area is 12. That is clear. In fact, if the two sides of the rectangle have lengths a and b then the area of the rectangle is ab . Even though you have known this since fourth grade, let's explore it a bit further.

Is this a law of nature that we humans have discovered, or is it a human convention? Does the area of a rectangle have to be the product of the two sides, or it is, because we have decided for it to be?

In fact, why is the area of a rectangle the product of the two sides? Let me first make a naive attempt at the meaning of area. We designate the area of a one by one square to be one--this will be our unit--and then the area of a rectangle is the number of these squares that fit into it exactly.

With this definition, it is clear why the area of a 3 by 4 rectangle is 12 since you can easily arrange 12 unit squares to create a 3 by 4 rectangle.

But what if the rectangle was $1/2$ by $1/2$? That is not difficult either. If we split the sides of our unit square into two equal parts, we can divide our unit square into four $1/2 \times 1/2$ little squares, and so the area of each of these is going to be a quarter.

In fact, the same type of argument can work with any rectangle whose sides are rational numbers--that is, the ratios of two integers. But what if the rectangle is a $\sqrt{2}$ by $\sqrt{2}$ rectangle? What is $\sqrt{2}$? It is approximately 1.41 and is a number that if you multiply it by itself, you get 2. Our formula says that the area of such a square is $\sqrt{2} \times \sqrt{2} = 2$. But why is that true?

In other words, why is it that we can fit two unit squares exactly in a square of side $\sqrt{2}$? This is not too hard either. Cut each of your two unit squares down their diagonal, get four right triangles, each with hypotenuse equal to $\sqrt{2}$, and rearrange them to get a square of side $\sqrt{2}$.

This works but should be a reason for concern. It is not clear how to extend that argument to other cases. In particular, if you right down the complete argument, it uses a fair amount of geometric facts, including the Pythagorean Theorem, which incidentally is Proposition 47 of Book I of Euclid's *Elements*.

So, for example, why is the area of a $\sqrt{3}$ by $\sqrt{3}$ rectangle equal to 3? In other words, how do we cut up three unit squares, rearrange them, and construct one square of side $\sqrt{3}$? This actually is not obvious at all. This particular problem, was first posed and solved by Abu'l Wafa Buzjani in Baghdad about 1,000 years ago. He worked on this problem as a response to applications. Artisans working with tiles wanted to know how to take a number of square tiles, cut them up, and make a single square tile from them. I urge you to try it as a little puzzle to see that it is not that clear how to do it.

Similarly, if you have a rectangle with sides π and $3\sqrt{2}$, the area of the rectangle is the product of these but I would be excited to see how many of you could actually give me a reason for it.

I could even ask you a seemingly simpler question--it actually is the same question--how do you multiply two irrational numbers? Each irrational number has an infinite non-repeating decimal expansion and so you can't even write down exactly what it is. How do you multiply two of them? Is it even possible to exactly multiply them?

I am not going to answer these basic questions, and I don't mean to suggest that you don't know anything and should go back to fourth grade or that you have to really understand Euclid before you are allowed to think about mathematics. In fact, the opposite. It took mathematicians a good 3,000 years to first realize that there is an issue here and then to resolve it. I don't have time to go through the fascinating history but I want to say a few words. In 300 BCE, Euclid wrote the all-time mathematics best seller, the *Elements*--it is still ranked in the top 10 among geometry books on Amazon.

Euclid talks a lot about triangles and rectangles but never talks about areas of rectangles, doesn't consider irrational numbers to be numbers, and never multiplies two irrational numbers. That is because he is very smart and doesn't like to talk about ideas he doesn't understand. A solid understanding of all the issues involved including the construction of real numbers and a complete answer to the questions that I raised today actually didn't appear until 1870s. And it didn't just happen over night.

Very important steps were slowly taken in the middle ages and in the Islamic lands. Without fully resolving the foundational issues, the concept of number was slowly expanded to include irrational numbers, the focus of mathematical reasoning was shifted from Geometry to Algebra, and approximations and numerical mathematics were taken seriously. These, in turn were essential for the subsequent development of mathematics including the development of calculus in Europe in the seventeenth century. After 200 years of creatively and productively using and extending calculus, certain problems about something called Fourier Series resulted in mathematicians going back to the drawing board and finally putting the concept of limit on a firm foundation. This was followed, some 40 years later, by a rigorous construction of real numbers and an answer to the questions posed here.

In other words, for hundreds of years mathematicians worked with ideas that they didn't really understand, and in fact their willingness to do so was essential in the actual development of the subject. Mathematics like other subjects didn't and doesn't grow linearly or neatly. It is usually only in retrospect and after exploring a lot of blind alleys that we can be confident that a certain path of research or reasoning has been productive.

A lot of people like math because it is a powerful tool in solving real world problems. I like it because of the ideas, the concepts, and the puzzles. Spending more than 2,000 years to clarify and understand one idea, that is what is fascinating to me.

You can certainly not worry about such things, and assume that your calculator knows enough so that it can provide a good enough answer to your problems. My point is: What if you are the calculator and someone pushes your buttons?

Now does it really matter that we understand things? I do not know the answer to that question. Astrophysicists tell us that the Universe will continue to expand forever and, in the long run, "will become a cold, dead wasteland with a temperature approaching absolute zero." So, in the very long run, it certainly doesn't matter.

In the very short term, it certainly matters to you, since you are in college, and maybe the purpose of college is to learn and understand things. In four years, you will be given a diploma and told that you are now ready to leave the Pomona bubble and enter the real world. Don't believe it for a second. You won't be ready for much. You will be ready for some things. You

will be ready for college at that point. Just like you are ready to nail high school right now. Your main goal in the next four years isn't to get ready. It is to find your passion, and you can do that only by going out of your comfort zones.

College is mostly a conversation and a chance to engage. We, the faculty, don't so much teach as provide a setting and a structure for you to grapple with ideas you don't understand. To engage in the conversation that is college, you don't need to already know but you do need other qualities. Curiosity, persistence, courage, a willingness to question, and to consider other perspectives is required.

Some of the conversation will happen in your classes, some when you are alone reading a book, or working on a problem set, and much of it will happen with your peers in hallways late at night. What I remember most vividly from my first semester at Oberlin, a long time ago, was the late night discussions. What did we discuss in the fall of 1973? The bloody CIA-coup in Chile that brought down Salvadore Allende, the Arab-Israeli war, and endless arguments about the relative merits of Bob Dylan vs. Jimi Hendrix.

But to have a conversation you need a partner, and it matters who you have the conversation with. At a residential liberal arts college like Pomona, the community that we bring together for a short four years affects what goes on here a great deal.

Pomona is a private institution but it has a very public mission. We are not here to maximize share holder returns or our own glory, or to strive to go up on the various college rankings. What we do here is to provide some of the best educational opportunities for a very small number of people.

If we are not going to just be a finishing school for the elite and the privileged, we have to constantly make sure that our doors are open to every sector of the society. Access to a Pomona education should not be reserved for any particular group. Now this is easier said than done. Over the last few decades, the number of applicants to our campus has grown tremendously, making us a very selective college. You are here because you have worked hard and because of your record. You should rightly be proud of your accomplishments, but it is also a fact that many others could be in your seats and do just as well. You also had the luck of the draw.

It is self-evident that there is no one measure that can be used to rank and to linearly order all applicants. Hence, it is silly to suggest that we should accept the "best" ones. Different people bring very different strengths to the table, and comparison is very difficult. In fact, how do you judge a whole person based on a file and maybe one interview. And how should you factor in the resources that a potential student has had at their disposal?

Should that matter?

This is particularly relevant in today's climate where public education is under attack everywhere. After one budget cut after another, the state of California, for example, spends less than \$8,000 annually per student in K-12 public schools. This leaves very little for anything but the bare minimums. The LA Unified School district, the second largest in the country, starting this year will have one counselor per 1,000 students. This doesn't affect students from privileged backgrounds as much since they can get the needed advising elsewhere, but it



certainly affects first-generation college students, those from economically disadvantaged backgrounds, and newly immigrant communities.

In fact, the rise in Pomona's applicant pool has been disproportionately from wealthy families. For Pomona to continue to aspire to its ideal of being accessible, it has to work very hard. And, we do have a long way to go. In last year's senior survey, 25% of our graduating seniors self reported as coming from families with annual household income of over \$250,000. Only about 2% of US households are in that income category but they provide 25% of our students. On the other hand, almost 50% of US households have an annual income of \$50,000 or less. Only about 15% of our students come from this half of the society.

I want to thank Pomona's trustees, administration, alumni and faculty for their unrelenting support of our need-blind admission policy and their commitment to student financial aid. I am also quite delighted that in the past few years Pomona has taken a number of steps to go beyond the usual avenues for identifying potential students. The Posse program, Pomona's own PAYS program, the Questbridge program, and the college's recent public support of the Dream Act all play an important part in making sure that we continue to keep making progress. I welcome each member of the class of 2014 to Pomona College and wish you good luck.

Thank you.