WiMSoCal Symposium

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Pomona College

Titles and Abstracts

Keynote Address by

KATHRYN LEONARD

CALIFORNIA STATE UNIVERSITY, CHANNEL ISLANDS

Modeling Shapes with Skeletal Fragments

Despite recent progress in image recognition, recognizing the shapes of objects in images continues to challenge researchers. Equally difficult are the tasks of decomposing a shape into its component parts, and determining which of those parts are similar to each other or to parts in another shape. We present a skeletal model for shapes, the Blum medial axis, and show how functions defined on the Blum axis can help with all three tasks. We also present results from a massive shape decomposition user study, showing how our automated shape decomposition compares to the human shape decomposition captured by the user study.

ABOUT THE SPEAKER:
Dr. Kathryn Leonard received her PhD from the Department of Mathematics at Brown University under the supervision of David Mumford. Such a destiny would have shocked her junior-year self, who was an English major arguing passionately against her math requirement. Fortunately for her, the University of New Mexico was unconvinced, thereby forcing her into the calculus class that began her love of math and led to a double major in math and English.
Dr. Leonard's interests arise from the mathematics of computer vision, including shape and texture modeling, which draws on tools ranging from

geometry to information theory. Her work has been recognized with a CAREER award from the National Science Foundation and a Henry L. Alder Award for Excellence in Teaching from the Mathematical Association of America. She has received over $1 million in grants from NSF and the W.M. Keck Foundation. Despite publishing papers in desirable journals, speaking at international conferences, and writing for Popular Science magazine, she still gets no respect from her cats.

Bahar Acu, USC
Symplectic Mapping Class Group Relations Generalizing the Chain Relation

In this talk, we will examine symplectomorphisms of higher dimensional symplectic manifolds by using fibered Dehn twists. In dimension 3, π_0 of the symplectomorphism group of a surface is equal to the mapping class group and hence can be studied entirely in terms of Dehn twists along simple closed curves. However, in higher dimensions, it is comparatively not as well understood. Despite the fact that many fibered Dehn twists cannot be expressed as a product of Dehn twists, we are still able to show that ∂W has an open book decomposition carrying a fibered Dehn twist which is symplectically isotopic to a product of $k(k-1)^n$ right-handed Dehn twists along Lagrangian spheres. Moreover, we show that this identification yields a generalization of the classical chain relation on surfaces. This is joint work with Russell Avdek.

Chloe Avery, UC Santa Barbara
Graceful Labelings of Armies of Caterpillars

The Graceful Tree conjecture is a problem in graph theory that, despite the efforts of many, is still open. The conjecture states that all graphs have a graceful labeling. A graceful labeling of a graph $G = \{V, E\}$ on n vertices is a way to give each vertex a distinct label from the set $\{1, 2, 3, ..., n\}$ such that if each edge is labeled with the absolute value of the difference between the two vertices that define each edge, the edge labels will all be distinct. Graceful labelings of graphs have many applications, including communication network design, optimal circuit layout, and additive number theory. I will briefly introduce graceful labelings of graphs and prove that a certain class of trees, called Armies of Caterpillars are indeed graceful.

Julie Bergner, UC Riverside
Fixed points of group actions on unitary partition complexes

The poset of nontrivial partitions of a finite set can be geometrically realized to obtain a topological space equipped with an action of the symmetric group. This space and its fixed point spaces by actions of subgroups of the symmetric group has been investigated in a series of papers by Arone, Dwyer, and Lesh and is expected to lead to a new proof of the Whitehead Conjecture. In joint work with Joachimi, Lesh, Stojanoska, and Wickelgren, we consider instead nontrivial decompositions of a finite dimensional complex vector space into orthogonal subspaces, which form a topological poset. We investigate its geometric realization and the fixed point spaces by actions of subgroups of the unitary group.

Cindy Blois USC

Functional Integral Representations for Quantum Many-Particle Systems

A "functional integral" is an integral where the domain of integration is represented as a space of functions. In this talk, we will see how such integrals can arise in quantum mechanics, noting that although they are generally not well-defined, they provide physical intuition. We will extend this discussion to many-body systems and outline the challenges of rigorously constructing functional integrals for these systems.

Jen-Mei Chang, CSU Long Beach

Increase student learning outcomes with flipped classrooms

Flipped learning is gaining traction in K-12 for enhancing students' problem solving skills at an early age; however, there is relatively little large-scale research showing its effectiveness in promoting better learning outcomes in higher education, especially in mathematics classes. In this talk, I will present some quantitative and qualitative findings on students' attitude changes and exam performance from a flipped introductory linear algebra class and a flipped Pre-Calculus class.
While it is encouraging to see that students in both flipped class performed superiorly in the overall comprehension of the content and experienced a change in their perspectives on the problem-solving process when compared to their counterparts, there are definitely psychological barriers when one decides to deviate from a lecture-based teaching paradigm, including fear of uncertainty and possible misconceptions of how classroom management might affect student learning. Hopefully the results of this talk may convince you that increased student engagement with advanced mathematics content in a supervised setting yield benefits that are worth the risk.

Courtney Davis, Pepperdine
A Clinically Parameterized Mathematical Model of Shigella Immunity to Inform Vaccine Design

We refine and clinically parameterize a differential equation model of the humoral immune response against Shigella, a diarrheal bacteria that infects 80-165 million people and kills an estimated 600,000 people worldwide each year. Using Latin hypercube sampling and Monte Carlo parameter estimation, we fit our model to human immune data from Shigella vaccine trials and a rechallenge study in which antibody and B-cell responses against Shigella's outer-membrane components were recorded. The clinically grounded model is used to mathematically investigate which key immune mechanisms and bacterial targets confer immunity against Shigella and to predict which immune components should be elicited to create a protective vaccine against Shigella. The model shows that, on average, humans would be highly symptomatic following a humoral immune response against only Shigella's outer-membrane components due to an uncontrolled infection of gut epithelial cells that is present across best-fit model parameterizations. Thus, our modeling results predict that a vaccine targeting only outer-membrane components will not be protective against Shigella. We also gain insight into why the vaccine candidate on average was not sufficiently efficacious. Using sensitivity analysis, we explore which model parameter values must be altered to prevent the destructive epithelial invasion by Shigella bacteria and identify key parameter groups as potential vaccine targets or immune correlates. This collaborative research underscores the need for a multifaceted approach in ongoing efforts to design an effective Shigella vaccine.

Ozlem Ejder, USC
Torsion Subgroups of Elliptic Curves

Let E be an elliptic curve defined over $Q(i)$ and F be the compositum of all quadratic extensions of $Q(i)$. I will determine 22 possibilities for the torsion subgroup structure of $E(F)$ and show that all but two groups on the list can be realized as a torsion subgroup of some elliptic curve over $Q(i)$. I will discuss the techniques we use to prove the result as well as the difficulties with the two groups which are of order 21 and 63.
This is a generalization of work of Laska, Lorenz and Fujita for the elliptic curves over Q.

Christina Eubanks-Turner, Loyola Marymount
Interlace Polynomials of the Friendship Graph and other Triangulated Graphs

Arratia, Bollobas, Coppersmith and Sorkin answer an important question related to DNA sequencing by introducing the interlace polynomial, a polynomial that represents the information gained from doing the "interlace" process on the graph. In this talk, I give an introduction of the interlace polynomial of a graph. I will also present results giving the interlace polynomial for the class of Friendship graphs, graphs that satisfy the Friendship Theorem given by Erdos, Renyi and Sos.

Sarafina Ford, UC Santa Barbara

Sign-Characteristic-Preserving Linearizations in DL(P)

A matrix polynomial $P(\lambda)$ is a polynomial whose coefficients are matrices with entries in an arbitrary field. / The polynomial eigenvalue problem consists of finding nonzero vectors x (called right eigenvectors) and scalars λ (called eigenvalues) such that $P(\lambda)x = 0$.
Matrix polynomials occur in areas including control theory and signal processing, so the computation of their eigenvalues and eigenvectors is an relevant and difficult problem. The standard approach this problem is using linearizations: matrix polynomials of degree 1 that share the eigenstructure of $P(\lambda)$.
Because of their structure, Hermitian matrix polynomials and their eigenvalues are of particular interest.
Attached to these eigenvalues is a set of signs called the sign characteristic, which establishes a relationship between the left and right eigenspaces of $P(\lambda)$. Thus, it is desirable to find linearizations that preserve this sign characteristic.
In our work, we consider the vector space $\mathbb{DL}(P)$ (constructed in 2006 by Mehl, Merhmann, Mackey and Mackey) of block-symmetric linear matrix polynomials, almost all of which are linearizations of $P(\lambda)$. Using techniques from indefinite linear algebra, we give a full characterization of the Hermitian linearizations in $\mathbb{DL}(P)$ that preserve the sign characteristic of $P(\lambda)$.
Strikingly, if a specific polynomial associated with the linearization is positive on the real spectrum of $P(\lambda)$, then the linearization preserves the sign characteristic of $P(\lambda)$. In future work, we will use this to determine large classes of sign characteristic preserving linearizations in $\mathbb{DL}(P)$ without requiring prior knowledge of the eigenvalues of $P(\lambda)$.

Talena Garman, Cal Poly Pomona

Chaotic-Signal Masking With Self-Synchronizing Systems

There are certain chaotic systems that exhibit rapid synchronization, which allows them to be used in communication applications, namely, chaotic-signal masking. We explored the Lorenz system described by Oppenheim and Cuomo and generated a digital program that successfully encrypted the authors sons recording, Hi, Mom. We then investigated two more systems that exhibit rapid synchronization Rssler and Pehlivan. We showed synchronization for the Rssler system, however we were unable to use it for chaotic-signal masking due to certain system criteria. Dr. Sundarapandian showed that the Pehlivan system will synchronize with an identical Pehlivan system with added nonlinear controllers, however he did not explore how this system could be used as a masking algorithm. Knowing that systems which exhibit rapid synchronization can be used in chaotic-signal masking, we were able to successfully use the Pehlivan system to encrypt the authors sons recording, Hi, Mom.

Kathleen Hake, UC Santa Barbara
Geometric Knot Theory and Symplectic Geometry

We can describe a n-sided polygon in 3-space as a point in 3n-space by listing in order the coordinates of its vertices. Then the space of n-sided polygons embedded in 3-space consists of a manifold in which points correspond to piecewise linear knots, while paths correspond to isotopies which preserve the structure of these knots. In addition, the space of polygons is a symplectic manifold. In this talk, we will discuss the topology of the space of hexagons as well as the symplectic structure.

Cymra Haskell, USC
Teaching Future Teachers

What kinds of mathematical knowledge do students need that are planning to teach K-12 mathematics? Is the mathematics they get in their undergraduate degree enough? In this talk we argue that it is not and describe a class we are developing at USC that is designed for students interested in becoming K-12 educators.

Jessica Jaynes, CSU Fullerton
Use of Orthogonal Array Composite Designs to Study Lipid Accumulation in a Cell-Free System

The development of clean, sustainable alternative energy sources is increasingly important. One promising alternative to depleting fuel reserves is algae based biodiesel fuel, which is both non- toxic and renewable. Despite the tremendous potential of algae-based biodiesel fuel, it has not yet been profitable due to the high cost per unit area of large cultivation. We propose a novel application of Orthogonal Array Composite Designs (OACDs) to optimize lipid production of a cell-free system for algae.
We begin with an initial screening experiment based on six chemicals using an OACD with 50 runs. Based on this experiment two chemical compounds were removed and a follow-up 25-run OACD with four

chemicals was performed. Our analysis shows that only three chemicals - nitrogen, magnesium and phosphate, are essential for lipid accumulation, and a range of optimum combinations of these three chemicals is identified. The lipid accumulation for these three chemical combinations is substantially higher in comparison to the commercial medium, which contains 16 chemicals and soil water. This leads to a reduced cost of the chemical medium and increased efficiency of biodiesel production from the algal-based cell-free system, which can be used to significantly expand the use of biodiesel as a viable alternative to fossil fuels.

Ezgi Kantarci, USC
Shifted Ribbon Tableaux

The shifted young tableaux give us a nice combinatorial tool to calculate Schur P functions, which are closely associated with the projective representations of the symmetric group. This talk will be about the ribbon tilings of the shifted diagrams, their quotients and their Schur P-positivity.

Ekaterina Merkurjev, USC
Graph MBO Method for Multiclass Segmentation of Hyperspectral Standoff Detection Video

We consider the challenge of detection of chemical plumes in hyperspectral image data. Segmentation of gas is difficult due to the diffusive nature of the cloud. The use of hyperspectral imagery provides non-visual data for this problem, allowing for the utilization of a richer array of sensing information. In this talk, we present a method to track and classify objects in hyperspectral videos. The method involves the application of a new algorithm recently developed for high dimensional data. It is made efficient by the application of spectral methods and the Nystro m extension to calculate the eigenvalues/eigenvectors of the graph Laplacian. Results are shown on plume detection in LWIR standoff detection.

Valerie Poynor, CSU Fullerton
Nonparametric Bayesian Inference for Mean Residual Life Functions in Survival Analysis

In survival analysis interest lies in modeling data that describe the time to an event. Informative functions can be obtained from the models distribution function. We focus on the mean residual life (mrl) function which provides the expected remaining life given survival up to a particular time. This function is of direct interest in reliability, medical, and actuarial fields. In addition to its practical interpretation, the mrl function characterizes the survival distribution. We review key properties of the mrl function and investigate its form for some common distributions. We develop general Bayesian nonparametric inference for mrl functions built from a Dirichlet process mixture model for the associated survival distribution. We discuss the importance of careful kernel selection to ensure desirable properties for the function. We advocate for a mixture model with a gamma kernel and dependent baseline distribution for the Dirichlet process prior. We study the empirical performance of our modeling technique in two simulation examples, a data set of two experimental groups, and a data set involving right censoring. We compare our results with an exponentiated Weibull model.

Puck Rombach, UC Los Angeles
Hat guessing games

Hat guessing games, in which players in a game have to guess the colour of the hat they are wearing based on the colours of hats of other players, are popular riddles amongst puzzle enthusiasts. Numerous variants have been considered by many authors. In the most popular variant, n players are assigned a hat uniformly and independently at random from s different colors. They can all see each other's hats, and they wish to maximize the probability that everybody guesses the colour of their own hat correctly. Much of the popularity of this puzzle is owed to the striking difference between the probability achieved by each player guessing independently and the one achieved by an optimal strategy; $(1/s)^n$ and $1/s$, respectively. This basic problem can be generalised to directed graphs where players can only see others along directed edges. We discuss some new results and open problems

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Lauren Ruth, UC Riverside
Understanding Jones' Index for Subfactors

We will summarize the proofs in V.F.R. Jones' landmark 1983 paper, "Index for Subfactors," which introduced a Galois theory for von Neumann algebras and pointed to new problems in operator theory. The classification of subfactors and the construction of examples realizing possible index values remain active areas of research today.

Deborah Tonne, CSU, Long Beach
A PDE Approach to Knot Theory: Models and Numerical Simulation

Knot theory is the study of closed, smooth curves embedded in three-dimensional space. An unknot is any knot that is ambiently isotopic to the standard embedding of the unit circle. Examples such as the Gordian unknot show that these embeddings can be quite complicated. A natural question, usually investigated using combinatorial techniques, is how to "optimally" simplify such complicated embeddings. Recently, PDEs derived from knot energies have been proposed as a means to continually deform a given knot to its simplified configuration. We propose and numerically investigate a PDE based model whose dynamics provably preserve isotopy class. Experiments on the Gordian unknot indicate the promise of the model to simplify any given embedding of the unknot.

Cindy Tsang, UC Santa Barbara
Galois Module Structure of Rings of Integers and Embedding Problems

Let K be a number field and let L/K be a finite Galois extension with group G. Let $\mathcal{O}_K$ and $\mathcal{O}_L$ be the rings of integers in K and L, respectively. An $\mathcal{O}_K$-basis of $\mathcal{O}_L$ is called a normal integral basis if the basis elements are Galois conjugates. Note that such a basis need not exist in general since $\mathcal{O}_L$ might not even be free over $\mathcal{O}_K$. The existence of such a basis is equivalent to $\mathcal{O}_L$ being free as an $\mathcal{O}_K G$-module. In this talk, I will study the $\mathcal{O}_K G$-module structure of $\mathcal{O}_L$ and explain how it is related to embedding problems.

Minaya Villasana, U Simón Bolívar and Pomona College
Evolutionary optimization of combined chemotherapy

This work studies evolutionary algorithms considered state of the art for real-valued coded solutions in the design of combined chemotherapy protocol. The treatment design can be formulated as an optimal control optimization problem. The objective is to minimize the tumor level during and at the end of treatment while keeping the immune system levels at acceptable levels. A comparison of three algorithms is considered, namely CMA Evolutionary Strategy, Differential Evolution and Particle Swarm Optimization. Comparison includes convergence properties, quality and diversity of solutions. Also different administration regimes are investigated with the two types of drugs considered: cytotoxic and cytostatic.

Yingying Wang, UC Santa Barbara
Introduction to p-adic L-functions

This will be an expository talk on p-adic L-functions. p-adic L-function is a analogue of Dirichlet L-funcitons. This talk will briefly introduce Dirichlet L-functions and it's properties and class number formula. Then I will give the outline of one construction of p-adic L-functions. We will discuss the values of them at s=1 and give a p-adic version of class nubmer formula and it's applicaitons.

Angela Wu, UC Los Angeles
The continuum random tree

The Continuum Random Tree (CRT) has been studied extensively since the eighties. In this talk, we will outline one way to construct the CRT, and describe some of its properties.

Nora Youngs, Harvey Mudd College
Neural codes, place fields, and convexity

Navigation is one of the most important functions of the brain. This year, the Nobel Prize in Medicine and Physiology was awarded for the discovery of place cells and grid cells, neurons which form vital pieces of the navigation system. Though the external observed correspondence of these neurons to 2D receptive fields has been carefully recorded and proven, the animal itself navigates the world without access to these maps. An important problem confronted by the brain is to infer what properties of a stimulus space can - in principle - be extracted from the stimulus space. This motivates us to define the neural ring and a related neural ideal, algebraic objects that encode the combinatorial data of a neural code. These objects can be expressed in a "canonical form" that directly translates to a minimal description of the receptive field structure intrinsic to the neural code. We also find that topological information about the stimulus space can be directly extracted from the neural ideal.